## Schedule

- 9:30-10:00 Registration.
- 10:00-10:05 **Prof Dominic Vella**, University of Oxford, Opening Remarks.
- 10:05-10:50 **Prof David Hobson**, University of Warwick, Prospect theory in a dynamic context.
- 10:50-11:10 **Neelima Neelima**, University of Edinburgh, *Higher order moment estimates for SPDEs and a generalization of existence of solutions to SPDEs under local monotonicity.*
- 11:10-11:30 Lawrence Middleton, University of Oxford, High-dimensional copula change-point models.
- 11:30-12:00 Coffee Break
- 12:00-12:20 Adam Barker, University of Reading, Fractal-dimensional properties of non-decreasing Lévy processes.
- 12:20-12:40 **Guangyu Xi**, University of Oxford, Aronson type estimate for parabolic equations with singular divergence-free drift.
- 12:40-13:00 Florian Klimm, University of Oxford, Rentian scaling for network analysis.
- 13:00-14:00 Lunch Break
- 14:00-14:30 Poster Session
- 14:30-14:50 Hadrien Montanelli, University of Oxford, Solving stiff PDEs on the sphere.
- 14:50-15:10 Niall Bootland, University of Oxford, Scalable two-phase flow solvers.
- 15:10-15:30 Maria Crespo Moya, Université de Montpellier, Asymptotic behavior of nematic liquid crystals.
- 15:30-15:50 Valentin Sulzer, University of Oxford, Electrochemical modelling of batteries for off-grid energy storage systems.
- 15:50-16:20 Coffee break
- 16:20-16:40 Doireann O'Kiely, University of Oxford, Stretching and buckling of thin viscous sheets.
- 16:40-17:25 **Prof Anne Juel**, University of Manchester, Ribbon curling via differential stretching of thin polymer sheets.
- 17:25-17:40 Closing Remarks/Awards
- 17:40-19:00 Pizza and Drinks in the Common Room

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# Plenary lectures

## 1. David Hobson, University of Warwick

Prospect theory in a dynamic context.

There are many problems in economics and medicine which involve choosing the time at which to sell a security or end a process. These are often modeled as optimal stopping problems in which the aim is to choose a stopping time to maximise the expected utility of some reward process. However, the predicted behaviour does not always match what we see in practice. Prospect theory provides an alternative description of agent behaviour which has found some support in static models. In this talk we discuss what happens when we combine optimal stopping with prospect theory.

## 2. Anne Juel, University of Manchester

Ribbon curling via differential stretching of thin polymer sheets

The procedure of curling a ribbon by running it over a sharp blade is commonly used when wrapping presents. Despite its ubiquity, a quantitative explanation of this everyday phenomenon is still lacking. We address this using experiment and theory, examining the dependence of ribbon curvature on blade curvature, the longitudinal load imposed on the ribbon and the speed of pulling. Experiments in which a ribbon is drawn steadily over a blade under a fixed load show that the ribbon curvature is generated over a restricted range of loads, the curvature/load relationship can be non-monotonic, and faster pulling (under a constant imposed load) results in less tightly curled ribbons. We develop a theoretical model that captures these features, building on the concept that the ribbon under the imposed deformation undergoes differential plastic stretching across its thickness, resulting in a permanently curved shape. The model identifies factors that optimize curling and clarifies the physical mechanisms underlying the ribbon's nonlinear response to an apparently simple deformation.

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## Student talks

## 1. Neelima Neelima, University of Edinburgh

Higher order moment estimates for SPDEs and a generalization of existence of solutions to SPDEs under local monotonicity.

We establish higher order moment estimates for solutions to nonlinear SPDEs by identifying the appropriate coercivity assumption. These are then used to prove existence of solutions to nonlinear SPDEs under local monotonicity conditions. Results in this direction by Röckner and Liu are generalized to allow derivatives in the operator acting on the solution under the stochastic integral. (jointly with D. Šiška).

## 2. Lawrence Middleton, University of Oxford

High-dimensional copula change-point models

Building on the wealth of work developed for copula models in time series, this work extends the high dimensional dynamic copula model of (Creal et al. 2015) to infer structural changes in the correlations in sets of time series. A novel particle Markov chain Monte Carlo inference scheme is developed to perform tractable inference in the model. The methodology performs well on a historical dataset of futures contracts as well as high dimensional synthetic data. The resulting methodology successfully infers the changepoint correlation structure corresponding to a number of significant macroeconomic events over the 2008 financial credit crisis.

### 3. Adam Barker, University of Reading

Fractal-dimensional properties of non-decreasing Lévy processes

We study the box-counting dimension of sets related to non-decreasing Lévy processes (subordinators). It was recently shown in [Savov, 2014] that almost surely  $\lim_{\delta \to 0} U(\delta) N(t, \delta) = t$ , where  $N(t, \delta)$  is the minimal number of boxes of size at most  $\delta$  needed to cover a subordinator's range up to time t, and  $U(\delta)$  is the subordinator's renewal function. The main result is a central limit theorem (CLT) for  $N(t, \delta)$ , complementing and refining work in [Savov, 2014]. Box-counting dimension is defined in terms of  $N(t, \delta)$ , but for subordinators we prove that it can also be defined using a new process obtained by shortening the original subordinator's jumps of size greater than  $\delta$ . This new process can be manipulated with remarkable ease in comparison to  $N(t, \delta)$ , and allows us to understand the box-counting dimension of a subordinator's range in terms of its LÃI'vy measure, significantly improving upon [Corollary 1, Savov, 2014]. Corresponding CLT and almost sure convergence results for the new process are then given for all subordinators with fractal structure (those with infinite Lévy measure).

References: [Savov, 2014] Mladen Savov. On the range of subordinators. Electronic Communications in Probability, 19:1-10, 2014.

## 4. Guangyu Xi, University of Oxford

Aronson type estimate for parabolic equations with singular divergence-free drift

In this talk, I will present the Aronson type estimate for a family of Parabolic equations with singular divergence-free drift in  $BMO^{-1}$  or Lebesgue spaces. Further, regularity of week solutions can be derived from such Aronson type estimates for  $BMO^{-1}$  or critical Lebesgue spaces. This estimate also guarantee the existence of the corresponding Diffusion processes. All these results relies critically on the divergence-free condition, which was motivated for applications in fluid dynamics. This talk is based on two recent joint works with Pro Zhongmin Qian arXiv:1704.02173 and arXiv:1612.07727.

## 5. Florian Klimm, University of Oxford

Rentian scaling for network analysis

Many systems can be represented as networks and show hierarchical organisation. One way to probe this structure is the determination of the Rentian Scaling, a method originating in the design of computer chips. We demonstrate the utility of this approach and explore network examples from neuroscience [1], biology [2], and public transportation [3]. The results indicate that the networks are embedded into space such that transport and information processing are efficient.

- [1] Klimm, F., Bassett, D. S., Carlson, J. M., & Mucha, P. J. (2014). Resolving structural variability in network models and the brain. PLoS Comput Biol, 10(3), e1003491.
- [2] Papadopoulos, L., Blinder, P., Ronellenfitsch, H., Klimm, F., Katifori, E., Kleinfeld, D., & Bassett, D. S. (2016). Embedding of biological distribution networks with differing environmental constraints. arXiv preprint arXiv:1612.08058.
- [3] Sperry, M. M., Telesford, Q. K., Klimm, F., & Bassett, D. S. (2016). Rentian scaling for the measurement of optimal embedding of complex networks into physical space. Journal of Complex Networks, cnw010.

## 6. Hadrien Montanelli, University of Oxford

Solving stiff PDEs on the sphere.

We present in this talk algorithms for solving stiff PDEs on the unit sphere with spectral accuracy in space and fourth-order accuracy in time. These are based on a novel variant of the double Fourier sphere method in coefficient space and implicit-explicit time-stepping schemes. A comparison is made against exponential integrators and it is found that implicit-explicit schemes perform better. Implementations in MATLAB and Chebfun make it possible to compute the solution of many PDEs to high accuracy in a very convenient fashion.

## 7. Niall Bootland, University of Oxford

Scalable two-phase flow solvers

Two-phase flows arise in many coastal and hydraulic engineering applications such as the study of coastal waves, dam breaking scenarios, and the design of coastal structures. However, modelling two-phase incompressible flow results in a variable coefficient Navier-Stokes system that is challenging to solve computationally. A crucial ingredient for a scalable numerical solution method is an efficient preconditioner for the discrete systems arising after linearisation. Here we build on existing techniques for single-phase flow, adapting the methodology to incorporate the two-phase nature of our governing equations.

## 8. Maria Crespo Moya, Université de Montpellier

Asymptotic behavior of nematic liquid crystals

We study the static equilibria of a simplified LeslieâAŞEricksen model for a unidirectional uniaxial nematic flow in a prototype microfluidic channel, as a function of the pressure gradient G and inverse anchoring strength, B. We numerically find multiple static equilibria for admissible pairs (G,B) and classify them according to their winding numbers and stability. The case G=0 is analytically tractable and we numerically study how the solution landscape is transformed as G increases. We study the one-dimensional dynamical model, the sensitivity of the dynamic solutions to initial conditions and the rate of change of G and G. We provide a physically interesting example of how the time delay between the applications of G and G can determine the selection of the final steady state.

### 9. Valentin Sulzer, University of Oxford

Electrochemical modelling of batteries for off-grid energy storage systems

"One of the greatest challenges in developing renewable energy sources is finding an efficient energy storage solution to smooth out the inherently fluctuating supply. One cheap solution is lead-acid batteries, which are used to provide off-grid solar energy in developing countries. However, modelling of this technology has fallen behind other types of battery; the state-of-the-art models are either overly simplistic, fitting black-box functions to current and voltage data, or overly complicated, requiring complex and time-consuming numerical simulations. Neither of these methods offers great insight into the chemical behaviour at the micro-scale.

In our research, we use asymptotic methods to explore the Newman porous-electrode model for a constant-current discharge at low current densities, a good estimate for real-life applications. In this limit, we obtain a simple yet accurate formula for the cell voltage as a function of current density and time. We also gain quantitative insight into the effect of various parameters on this voltage. Further, our model allows us to quantitatively investigate the effect of ohmic resistance and mass transport limitations, as a correction to the leading order cell voltage. Finally, we explore the effect on cell voltage of other secondary phenomena, such as growth of a discharge-product layer in the pores and reaction-induced volume changes in the electrolyte."

10. Doireann O'Kiely, University of Oxford

Stretching and buckling of thin viscous sheets

Thin glass sheets are used in smartphone, battery and semiconductor technology, and may be manufactured by first producing a relatively thick glass slab and subsequently redrawing it to a required thickness. In this latter redraw process, the sheet is fed into a furnace where it is heated and stretched so that its thickness decreases. However, undesirable features such as non-uniform thickness and out-of-plane ripples commonly form, and the resulting sheets are not suitable for purpose. In this talk I will present a mathematical model for the glass sheet redraw process and discuss the formation of ripples in the sheet during drawing.

## Posters

#### Jonathan Harrison, University of Oxford

The impact of temporal resolution on parameter inference for biological transport models

When collecting time series data of biological transport processes, it is generally necessary to observe the system at discrete time points, for example via an imaging experiment. This can introduce errors when the motion of an observed object is approximated with discrete steps. We study the impact of collecting data at different temporal resolutions on parameter inference for biological transport models. In this work, we have performed exact inference for velocity jump process models in a Bayesian framework. This allows us to obtain estimates of the model parameters, including the turning rate and noise amplitude, based on noisy observations of this transport process. We show sensitivity of these estimates to changes in time discretisation and noise amplitude. For a fixed photon budget, our results suggest that better estimates of parameters can be obtained when imaging more frequently with more noise than imaging sparsely with low noise.

Victoria Pereira, University of Oxford

 $Multiphase\ pipeflow$ 

An overview of how to model an oil well through multiphase modelling.

Lindon Roberts, University of Oxford

Derivative-free optimisation for least-squares problems

Derivative-free optimisation (DFO) algorithms are a category of optimisation methods for situations when one is unable to compute or estimate derivatives of the objective function. The need for DFO arises in applications where techniques such as finite differencing or algorithmic differentiation are inaccurate or impractical, such as when the objective has noise (e.g. Monte Carlo simulations in finance) or is very expensive (e.g. climate simulations). In this poster, I present a simplification of state-of-the-art DFO methods for least-squares problems (relevant for problems such as parameter fitting), which gives comparable performance but with substantially reduced runtime.

Ian Roper, University of Oxford Silicon anodes in lithium-ion batteries

Lithium-ion batteries are becoming increasingly common in everyday life with mobile phones and laptops being essential to everyday life and electric cars coming into the mainstream market. The demand for high power and long-lasting batteries has caused vast research to be put into finding new materials for anodes and cathodes. One promising anode material is silicon, as this has the largest volumetric and gravimetric capacities for lithium ions currently known. However, upon lithiation, silicon increases volume by up to 300%, causing high stresses on the battery as well as cracking in the anode itself. Many anode material companies, including Nexeon (with whom this project is in collaboration), have developed methods of mitigating this expansion and reducing the stresses. These include using micro-scale particles (known as secondary particles) or nanofibres to decrease the stresses and using graphite shells around a silicon core to constrain the expansion. We look to mathematically model this expansion using solid mechanics to predict the behaviour of the anode using different designs of secondary particles. This will potentially shed light on an optimal anode design for future lithium-ion batteries.

We use a homogenisation technique to relate the expansion in a secondary particle to the expansion of the whole anode. We also analyse the micro-stresses occurring in the secondary particles for different geometries. We extend the current model by Chakraborty, Chapman, Goriely and Please (who worked with unimaterial designs) to analyse the use of silicon core/graphite shell structures and investigate the effect of different core sizes, central voids and porosity. We find that the graphite is too weak to constrain solid silicon, which appears to expand freely. Further, when a central void is used in the middle of the silicon core, the anode material continues to expand outwards as opposed to inwards, thus ignoring the void. Finally, we investigate designs using porous silicon layers and find that the expansion can be slightly mitigated without sacrificing the high capacity of silicon.

**Thomas Roy**, University of Oxford *Preconditioning in reservoir simulation* 

Oil and gas reservoir simulation entails solving systems of coupled nonlinear PDEs describing the flow of fluids (typically oil, water and gas) through porous media. Most of the computational time is spent on solving the resulting linearised systems with a preconditioned Krylov subspace method. The main preconditioning techniques still use the approach introduced by Wallis in 1983, the Constrained Pressure Residual (CPR) preconditioner. In this method, an accurate preconditioner is used for the pressure subsystem, in combination with a cheaper preconditioner for the full system. In this poster, we discuss the details of these two-stage preconditioners and how to treat the coupling between the pressure and the other variables.

## Fabian Ying, University of Oxford

Reducing congestion in supermarkets with queuing networks

Reducing congestion inside stores (e.g., supermarkets) is of great interest to many retailers; congestion affects customer experience, and may delay the fulfillment of online orders. We model stores as planar graphs in which nodes represent zones, and edges connect the nodes of neighboring zones. Customers traverse through the graph via the edges, and they queue to be served at each node. Once they have been served, they visit the next node. This approach allows us to apply standard results from queuing theory to find queue sizes and other quantities of interest.

Our results suggest that queuing networks can be used successfully to give insight for the design of better stores. We also investigate features of *optimal networks* and conjecture the optimal network topology for an arbitrary network size.