The 10th Oxford University SIAM-IMA Student Chapter Conference

Mathematical Institute, University of Oxford

2nd of May 2018

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Schedule

9:30-10:00 Registration in front of lecture theatre **L3**.

10:00-10:05 **Prof Dominic Vella**, University of Oxford, Opening Remarks.

10:05-10:50 **Dr Petra Vertes**, University of Cambridge, *Networks in Neuroscience.*

10:50-11:10 **Fabian Ying**, University of Oxford, *Minimising congestion in supermarkets.*

11:10-11:30 **Roxana Pamfil**, University of Oxford, *Communities in multilayer product-purchase networks.*

11:30-12:00 Coffee Break

12:00-12:20 **Matthew Butler**, University of Oxford, *A bio-inspired design for a switchable elastocapillary adhesive.*

12:20-12:40 **Danny Groves**, Cardiff University, *Droplet spreading, chemically treated surfaces and mathematics.*

12:40-13:00 **Joe Bishop**, Cardiff University, *The analysis of bending thin periodicity perforated plates.*

13:00-14:00 Lunch Break

14:00-14:30 Poster Session

14:30-14:50 **Joseph Field**, University of Oxford, *Compressed Sensing Reconstruction of Dynamic X-ray Imaging.*

14:50-15:10 **Alexander Safar**, Cardiff University, *Debonding and stretching in biogenic cellular structures.*

15:10-15:30 **Adam Barker**, University of Reading, *Markov processes with constrained local time.*

15:30-15:50 **Seungchan Ko**, University of Oxford, *Finite element approximation of incompressible chemically reacting non-Newtonian fluids.*

15:50-16:20 Coffee break

16:20-16:40 **Rachel Philip**, University of Oxford, *Droplet breakup in a deep-sea oil spill.*

16:40-17:00 **Clint Wong**, University of Oxford, *Fluid flow through vegetation.*

17:00-17:45 **Dr James Sprittles**, University of Warwick, *Nanoscale free surface flows: modelling beyond Navier-Stokes-Fourier.*

17:45-18:00 Closing Remarks/Awards

18:00-19:30 Pizza and Drinks in the Common Room

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Plenary lectures

1. **Petra Vertes**, University of Cambridge
   Networks in Neuroscience.

   In this talk I will give a brief overview of how network science has been applied in recent years to advance our understanding of the human brain. I will focus particularly on largescale networks derived from neuroimaging data and their application to the study of brain and mind disorders. I will discuss recent efforts to gain a more mechanistic understanding of disease processes by (1) generative modelling of human brain networks (2) integration of neuroimaging data with microscopic atlases of the human brain, containing detailed information on cellular characteristics or on regional gene expression. I will also discuss parallel efforts to understand brain organization at the neuronal scale both in silico and in simpler organisms such C. elegans. These provide a unique test-bed for developing and validating novel approaches, such as the application of control theoretic principles to the nervous system. Throughout the talk, I will highlight the many challenges and exciting opportunities that lie ahead.

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2. **James Sprittles**, University of Warwick
   Nanoscale free surface flows: modelling beyond Navier-Stokes-Fourier

   Understanding the behaviour of liquid-gas interfaces at the micro and nano scale is key to a myriad of phenomena, ranging from the formation of clouds through to 3D printing. Accurate experimental observation of such phenomena is complex due to the small spatio-temporal scales of interest and, consequently, mathematical modelling and computational simulation become key tools with which to probe such flows. As the characteristic scales of interest become comparable to microscopic scales, for a gas the mean free path and for a liquid the molecular diameter, the basic Navier-Stokes-Fourier (NSF) paradigm no longer provides an accurate description of the flow physics. However, microscopic models such as the kinetic theory of gases or molecular dynamics (MD) of liquids become computationally intractable for many flows of practical interest. In this talk I will consider a number of approaches to going beyond NSF, whilst remaining in a continuum framework that allows for efficient computation and clear analysis of results. I will consider two specific free surface flows which are influenced by nanoscale physics, namely, the breakup of liquid nanojets, where MD will provide a benchmark, and the role of gas nano films trapped under impacting of liquid drops, where we can compare directly to recent experimental observations. In each case a model going beyond NSF will be developed and its implications will be discussed.

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Student talks

1. **Fabian Ying**, University of Oxford
Minimising congestion in supermarkets.

   In this talk, I will present our work on modelling customer movement in supermarkets with application to reducing congestion inside supermarkets. Identifying and potentially reducing congestion within a supermarket both enhances the shopping experience for customers and reduces fulfilment time and costs for online orders. We apply human mobility models to fit anonymised customer trip distribution that were collected through Tesco's Scan-As-You-Shop scheme. These mobility models have been mainly used on large spatial scales (e.g., on a inter-country or inter-city level), but we show that several of these models can successfully predict customer mobility inside supermarkets. Finally, we apply a simple simulated annealing algorithm to find better store layouts with less congestion.

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2. **Roxana Pamfil**, University of Oxford
Communities in multilayer product-purchase networks

   A network model of shopping activity consists of customers connected to products that they previously purchased. Densely connected clusters or "communities" in these networks reveal customers with similar preferences and the products that they buy the most. It is also possible (and sometimes more insightful) to detect communities in temporal networks (in which connections vary through time), multilevel networks (which can encode product hierarchies), and other types of multilayer networks. In this talk, I will show how to perform community detection in these networks in a statistically-grounded way, without the need for parameter tweaking.

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3. **Matthew Butler**, University of Oxford
A bio-inspired design for a switchable elastocapillary adhesive

   Many species of insects adhere to vertical and inverted surfaces using footpads that secrete thin films of a mediating fluid that bridge the gap between the foot and the target surface. The precise role of this liquid is still subject to debate, but it is thought that the contribution of surface tension to the adhesive force may be significant. It has also been observed that the footpad is soft, suggesting that capillary forces might deform its surface. Inspired by these physical ingredients, we study a model problem in which a thin, deformable membrane under tension is adhered to a flat, rigid surface by a liquid droplet. We find that the presence of elastic deformation significantly enhances the adhesion force compared to a rigid footpad, and that there are two types of equilibrium state, one of which has substantially greater adhesion force. The equilibria can be controlled via two key parameters that depend on the imposed separation of the foot and target surface, and the tension applied to the membrane. We confirm these findings experimentally and show that the system may transition rapidly between two states as the two parameters are varied. This suggests that different strategies may be used to adhere strongly and then detach quickly.

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4. **Danny Groves**, Cardiff University
   Droplet spreading, chemically treated surfaces and mathematics

   Whether it's the raindrops on your car's windshield, or the water running off a plant leaf, we frequently encounter the rich physics behind droplet motion and yet remain unaware of its impact on the modern world. For many decades this class of phenomena has fascinated academics from all areas of science and engineering, furthermore, this interest has brought the development of new technologies such as ink-jet printers, and the design of self-cleaning surfaces. From a mathematical point of view, this situation leads to fourth order non-linear partial differential equations, where solutions rely on the use of powerful numerical tools. Therefore, emphasis is placed on theoretical methods such as matched asymptotic expansions which reduce the complexity of the full equations to form approximate models. These derived simpler models retain the key mechanisms that drive droplet motion, allowing us to further explore the physics behind droplet spreading, as well as obtain vast speedups to the full numerical calculations where a sample simulation has shown to be 250,000 times faster than a supercomputer calculation.

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5. **Joe Bishop**, Cardiff University
   The analysis of bending thin periodicity perforated plates

   In the modern scientific world many complex problems rely on powerful computing tools to determine solutions, however, as advanced as computing power may be, there still remains limitations. The numerical modelling of microstructures can be quite costly even for simple simulations, since the numerical grid required needs to be small enough in order to resolve each individual âĂIJmicro-cellâĂİ. This is the nature of our problem where we begin from a non-linear elasticity setting and consider the bending energy of a thin elastic plate which is periodically perforated throughout the domain. We derive the gamma-limit of the functional that describes the bending energy of the plate (as the thickness of the plate and the period of the perforations tend to zero) through simultaneous homogenisation and dimension reduction. Therefore obtaining a macroscopic model that retains information of the microstructure.

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6. **Joseph Field**, University of Oxford
   Compressed sensing reconstruction of dynamic x-ray imaging

   Medical imaging is a key diagnostic tool, and to reduce the amount of radiation that a patient may be subjected to there can be a strong incentive to consider image reconstruction from incomplete sets of measurements. With this in mind, we can formulate the imaging process as a compressed sensing problem. We focus on a compressed sensing approach for digital tomosynthesis (DTS), in which three-dimensional images are reconstructed from a set of two-dimensional X-ray projections. In particular, we are interested in image reconstruction for measurements affected by motion, where there is no assumed knowledge of the movement in question. We present a scheme for bodies undergoing simple linear motion, showing that reconstructions are more accurate than those obtained if the body was stationary.

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7. **Alexander Safar**, Cardiff University
   Debonding and stretching in biogenic cellular structures

   Debonding and stretching in biogenic cellular structures: Cellular tissues such as apples, pears and potatoes are a collection of fluid filled parenchyma cells bound together by inter-cellular cohesion. In a ripe and juicy apple, fluid is released from cells as the cell wall ruptures (cell bursting). In overripe or cold-stored fruit the strength of the inter-cellular cohesion decreases and the cell wall strength increases, such that it takes less energy to separate cells than to burst. This study has two parts, first involves the phenomena of cell separation, or debonding, as it is key in explaining the behaviour of fruit and legumes during storage or cooking, and is decisive for the quality of food products. Numerical models are used to provide evidence of how the cell wall, cell contents and inter-cellular cohesion contribute to cell debonding in soft fruits and tissues. Particular focus is given to shear deformation as this has been largely neglected in literature. The second part involves a multi-scale framework which relates the deformation and stress of a whole structure, to the cell wall (or vice versa). Often cell wall characteristics are not known explicitly and can now be inferred from macroscopic tests. The choice of hyperelastic material for the cell wall is arbitrary within the framework but a new Bayesian selection criteria for fitting material models to experimental data is discussed.

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8. **Adam Barker**, University of Reading
   Markov processes with constrained local time

   We study the problem of a Markov process conditioned so that its local time must grow slower than a prescribed function. Building upon recent work on Brownian motion with constrained local time in [1], we study the problem for a large class of Markov processes. We find a necessary and sufficient condition for transience/recurrence of the conditioned process, and also explicitly determine the distribution of the conditioned (inverse) local time. In the transient case, we explicitly determine the law of the conditioned Markov process. In the recurrent case, we further determine the "entropic repulsion envelope", which formally characterises how the process is affected by "entropic force" (the tendency of a system to increase its entropy). This work is theoretical, but is related to problems in polymer physics in which a long polymer chain is modelled by a random process which is in some sense "weakly self-avoiding" (constraining the local time corresponds to "weak" self-avoidance).

References: [1] Kolb, M., Savov, M. Transience and recurrence of a Brownian path with limited local time, Ann. Probab., 2016

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9. **Seungchan Ko**, University of Oxford
Finite element approximation of incompressible chemically reacting non-Newtonian fluids

We consider a system of nonlinear partial differential equations modelling the motion of an incompressible chemically reacting non-Newtonian fluid. The governing system consists of a convection-diffusion equation for the concentration and the generalized NavierâĂŞStokes equations, where the viscosity coefficient is a power-law type function of the shear-rate, and the coupling between the equations results from the concentration-dependence of the power-law index. This system of nonlinear partial differential equations arises in mathematical models of the synovial fluid found in the cavities of movable joints. We construct a finite element approximation of the model and perform the mathematical analysis of the numerical method. Key technical tools include discrete counterparts of the Bogovskii operator, De Giorgi type regularity theorem, and the AcerbiâĂŞFusco Lipschitz truncation of Sobolev functions, in function spaces with variable integrability exponents. This is joint work with Endre Suli and Petra Pustejovska.

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10. **Rachel Philip**, University of Oxford
Droplet breakup in a deep-sea oil spill

In a deep-sea oil spill a broken pipe, hundreds of metres below the sea surface, releases a highly turbulent jet of oil into the water. The jetâĂŹs vigorous turbulent mixing breaks up oil drops in the jet and produces smaller drops. These smaller oil droplets can then be eaten by microbes in the sea, and thus are removed from the water column before reaching the surface. To understand the factors affecting droplet breakup, we must examine the disparate macroscopic and microscopic scales which characterise this problem. On the large scale, we look at the turbulent jet and on the small scale, the droplets which are being broken up. We then unify these scales, combining similarity solutions for the jetâĂŹs velocity and energy dissipation with droplet population models. This approach produces a new mathematical description of the spatial variation of droplet size distribution throughout the jet. The behaviour is described by a superposition of similarity solutions, each of which is given by solving an eigenvalue problem. We use this model of the droplet size distribution, varying the initial drop sizes and droplet breakup parameters, to provide insights into the factors affecting droplet breakup in deep-sea oil spills.

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11. **Clint Wong**, University of Oxford
Fluid flow through vegetation

The study of fluid flows interacting with vegetative structures presents a significant theoretical and numerical challenge on account of its inherently multi-scale nature. Consider, for example, the case of fluid passing over a submerged vegetative bed or layer. Even in the case of inelastic vegetation, the flow characteristics can be highly dependent on the arrangement and configuration of the bed; here, strong shear effects along the top of the vegetation can trigger vortices, analogous to the case of the Kelvin-Helmholtz instability. In this talk, we will discuss the mathematical modelling of this fluid-structure problem, which now incorporates elastic deformation of the vegetation. A stability analysis of this problem suggests new insights into how the deformation of vegetation can affect vortex generation and the stability threshold.

Posters

Alex Bradley, University of Oxford
Bendotaxis – A novel droplet transport mechanism

We demonstrate bendotaxis, a novel droplet transport mechanism at small scales. A combination of bending and capillarity in a thin channel results in a pressure gradient that, in turn, results in the spontaneous movement of a liquid droplet. Surprisingly, the direction of this motion is always the same, regardless of the wettability of the channel. We use a combination of experiments at a macroscopic scale and a simple mathematical model, to study this motion, focussing in particular on the time scale associated with the motion. We suggest that bendotaxis might be a useful means of droplet transport, for example in developing self-cleaning surfaces, and discuss the implications of our results for such applications.

Bernabe Gomez Perez, Cardiff University
Mathematical modelling of tsunamis

Tsunamis are natural hazards that can be devastating in real World. Therefore, there is a need for an efficient and reliable early Tsunami warning system. Beside the Tsunami acoustic waves are generated, which can leave measurable bottom pressure signals far from the fault and those can be recorded by distant hydrophones.
We aim at developing an automatized program capable to retrieve the characteristics of a certain earthquake through acoustic wave pressure recordings. It has been shown that for a great distance between the source and the observation point, the acoustic waves are decoupled from gravity waves, so that we can study them separately and they travel at speed of sound in water which could be more than 8 times faster than Tsunamis speed.
The three dimensional wave equation is solved with standard boundary conditions. Tsunamis in the ocean are generated often by submarine earthquakes of slender rectangular shape, where slender body theory can be applied, leading to the use of multiple scales.
We modelled several slender shape earthquakes with the purpose of analysing the generated signals and retrieve the earthquake characteristics efficiently, which are the location, eruption time, uplift displacement and speed and earthquakes geometry.
Once all the characteristics are obtained the potential Tsunami is propagated and the risk is assessed.

Camille Poignard, ICMC São Carlos, USP
The effects of structural perturbations on the dynamics of networks

We study the effects of structural graphs perturbations of diffusive networks (represented by coupled ODEs) on their ability to synchronize: that is to say, we look at the hinderance or reinforcement of synchronizability when we add directed links of small weights to a directed network or when we perturb the weights of some of its existing links. This problem is of interest for real-world networks: for instance, given a power grid, people are interesting in understanding the effects of adding new connexions on the occurence of blackouts. Studying the synchronizability of networks does not only concern energy grids: in neurology for instance, synchronization of subcortical brain areas has been investigated a lot, since it is strongly believed to be the origin of motor diseases such as Dystonia and Parkinson. Our method is mainly based on the theory of perturbations of eigenvalues, and on our very recent structural genericity properties obtained for Laplacian graphs spectra. This work focuses both on undirected networks and on directed ones: in the undirected case we can rely on graph partitioning theory developed by Fiedler. In the directed case there is no such graph partitioning theory: for this situation we will restrict our study to weakly connected networks presenting a master-slave configuration. This work is based on 2 publications: one published in SIAP in January 2018, and the other one submitted to Nonlinearity.

Danny Groves, Cardiff University
Droplet spreading, chemically treated surfaces and mathematics

Whether it's the raindrops on your car's windshield, or the water running off a plant leaf, we frequently encounter the rich physics behind droplet motion and yet remain unaware of its impact on the modern world. For many decades this class of phenomena has fascinated academics from all areas of science and engineering, furthermore, this interest has brought the development of new technologies such as ink-jet printers, and the design of self-cleaning surfaces. From a mathematical point of view, this situation leads to fourth order non-linear partial differential equations, where solutions rely on the use of powerful numerical tools. Therefore, emphasis is placed on theoretical methods such as matched asymptotic expansions which reduce the complexity of the full equations to form approximate models. These derived simpler models retain the key mechanisms that drive droplet motion, allowing us to further explore the physics behind droplet spreading, as well as obtain vast speedups to the full numerical calculations where a sample simulation has shown to be 250,000 times faster than a supercomputer calculation.

Florian Klimm, University of Oxford
Modular structure in temporal protein interaction networks

Protein interaction networks (PINs) allow the representations of biological processes in cells and analysing them with tools of machine learning can help us understanding them. Because cells are dynamic and adaptive, these processes change over time. One example of adaptive regulation is the change of gene expression, which may occur at very different time scales: responses to environmental signals take minutes, and developmental changes take days in nematodes as *C. elegans* and years in humans. This change in protein expression results in an altered protein abundance in an organism.

Thus far, research has focused either on the static PIN analysis or the temporal nature of gene expression. By analysing temporal PINs using multilayer networks, we want to link these efforts. The analysis of temporal PINs with machine learning gives insights into how proteins, individually and in their entirety, change their biological functions. We present a general procedure how to integrate temporal gene expression information with a monolayer PIN to a temporal PIN and how to detect a modular structure using multilayer modularity maximisation, which is one way to cluster nodes in networks.

The nematode *C. elegans*' PIN changes during developmental stages. Using gene ontology (GO) terms, we connect this structural change with a reorganisation of biological functions. To our knowledge, our results represent the first direct identification of dynamic modular structure in PINs, despite having been hypothesised more than a decade ago.

Melanie Beckerleg, University of Oxford
Machine learning and matrix completion for bioactivity prediction

In-silico methods are a vital part of identifying potential drug compounds to target complex diseases. Machine learning methods are used to predict bioactivity from sparse experimental data, however they can be difficult to interpret. We apply techniques from database tiling literature, approximating databases using binary low rank matrices, identifying subgroups of proteins and compounds that demonstrate similar bioactivity behaviour, and compare to a Random Forest algorithm adapted for matrix completion.

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